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Selection rules for polymers and quasi one-dimensional crystals: III. Kronecker products for the line groups isogonal to $\mathbf{D}_{n\mathrm{d}}$

Ivan Božović†, Nataša Božović‡ and Milan Damnjanović§

† Institute of Physics, PO Box 57, 11000 Belgrade, Yugoslavia
‡ Department of Mathematics, Faculty of Science, The University, PO Box 550, 11000 Belgrade, Yugoslavia
§ Department of Physics, Faculty of Science, The University, PO Box 550, 11000 Belgrade, Yugoslavia

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Abstract. Line groups are symmetry groups of polymers and quasi one-dimensional solids. The reduction coefficients for the Kronecker products of irreducible representations of these groups give selection rules for various physical processes in polymers, and they are tabulated here for all the line groups isogonal to $\mathbf{D}_{n\mathrm{d}}$ ($n = 1, 2, \ldots$) point groups. The conservation laws for the corresponding quantum numbers (quasi-momentum, quasi-angular momentum and certain parities) are obtained.

1. Introduction

A knowledge of selection rules for various scattering processes in polymers and quasi one-dimensional solids facilitates the understanding of their electronic properties, which have attracted great attention recently (Seymour 1981, Miller 1982). The reduction coefficients for the Kronecker products of the irreducible representations (reps) of the line groups, which describe the spatial symmetries of these physical systems, are thus needed. In the preceding two papers of the series (Damnjanović *et al* 1983, 1984, to be referred to as I and II, respectively), such coefficients have been tabulated for the line groups isogonal to $\mathbf{C}_n$, $\mathbf{C}_{n\mathrm{v}}$, $\mathbf{C}_{n\mathrm{h}}$, $\mathbf{S}_{2n}$ and $\mathbf{D}_n$. Here we deal with the line groups isogonal to $\mathbf{D}_{n\mathrm{d}} = \mathbf{C}_{n\mathrm{v}} + \mathrm{U}_\mathrm{d}\mathbf{C}_{n\mathrm{v}}$, $n = 1, 2, \ldots$, where $\mathbf{C}_{n\mathrm{v}} = \mathbf{C}_n + \sigma_\mathrm{v}\mathbf{C}_n$ and where $\mathbf{C}_n$ is generated by C_n, the rotation through $2\pi/n$ around the z axis; σ_v is the reflection in the xz plane, $\mathrm{U}_d = \mathrm{C}_{2n}U$ and U is the rotation through π around the x axis. The same method is utilised as in I and II; the specific difficulties here are that the line groups under study are two-step extensions, and that the number of the reps to be considered is larger. This fact also made necessary a somewhat different organisation of the tables; for the same reason the short rep symbols are introduced (in § 2).

The line groups isogonal to $\mathbf{D}_{n\mathrm{d}}$ are $\mathbf{L}\bar{n}\boldsymbol{m}$ ($n = 1, 3, \ldots$), $\mathbf{L}(\overline{2n})\mathbf{2}\boldsymbol{m}$ ($n = 2, 4, \ldots$), $\mathbf{L}\bar{n}\boldsymbol{c}$ ($n = 1, 3, \ldots$) and $\mathbf{L}(\overline{2n})\mathbf{2}\boldsymbol{c}$ ($n = 2, 4, \ldots$). The first two families contain symmorphic line groups and the latter two contain non-symmorphic ones. For each family we give the character table to define the rep symbols and to specify the ranges of their quantum numbers; more details can be found in Božović and Vujičić (1981). Then for every pair D, D' of the reps we specify the irreducible components of the Kronecker product $D\times D'$. The listing is triangular, since $D\times D'\sim D'\times D$. Each row is identified by a boldfaced rep symbol.

2. Notation

In this paper, certain frequently occuring expressions involving the quantum numbers k, k', m and m' are abbreviated as follows:

$$u=q-m \qquad v=\begin{cases} m'-m, \text{ if } m-m'\in[\frac{1}{2}(3-n), -1] \\ m-m', \text{ if } m-m'\in[1, \frac{1}{2}(n-3)] \end{cases}$$

$$w=\begin{cases} m+m', & \text{if } m+m'\in[2, \frac{1}{2}(n-1)] \\ n-m-m', & \text{if } m+m'\in[\frac{1}{2}(n+1), n-1] \end{cases}$$

$$x=k-k', \text{ where } k-k'\in(0, \pi), \qquad y=\begin{cases} k+k', \text{ if } k+k'\in(0, \pi) \\ 2\pi-k-k', \text{ if } k+k'\in(\pi, 2\pi) \end{cases}$$

$$z=\pi-k.$$

Certain statements are abbreviated also by numbers or letters:

(1) $\leftrightarrow$ $(m-m'\neq 0$ and $m+m'\neq 0)$ (2) $\leftrightarrow$ $(m-m'=0$ and $m+m'\neq 0)$;

in the case when $n=2q=2, 4, 6, \ldots$

(3) $\leftrightarrow$ $(m-m'\neq 0$ and $m+m'=q)$ and (4) $\leftrightarrow$ $(m-m'=0$ and $m+m'=q)$.

Similarly,

(a) $\leftrightarrow$ $(k-k'>0$ and $k+k'\neq\pi)$ (b) $\leftrightarrow$ $(k-k'=0$ and $k+k'\neq\pi)$
(c) $\leftrightarrow$ $(k-k'>0$ and $k+k'=\pi)$ (d) $\leftrightarrow$ $(k-k'=0$ and $k+k'=\pi)$
(e) $\leftrightarrow$ $(k-k'>0$ and $k+k'<\pi)$ (f) $\leftrightarrow$ $(k-k'>0$ and $k+k'>\pi)$
(g) $\leftrightarrow$ $(k-k'=0$ and $k+k'<\pi)$ (h) $\leftrightarrow$ $(k-k'=0$ and $k+k'>\pi)$

3. Results

3.1. *The symmorphic line groups* $\mathbf{L\bar{n}m}$ *($n=1, 3, 5, \ldots$) and* $\mathbf{L(\overline{2n})2m}$ *($n=2, 4, 6, \ldots$).*

Table 1. The characters of the reps of the line groups $\mathbf{L\bar{n}m}$ ($n=1, 3, 5, \ldots$) and $\mathbf{L(\overline{2n})2m}$ ($n=2, 4, 6, \ldots$). Here $s=0, 1, \ldots, n-1$; $t=0, \pm1, \ldots$; $\alpha=2\pi/n$; the translation period is taken for the length unit so that $k\in(0, \pi)$; m takes on all integral values from the interval $[1, \frac{1}{2}(n-1)]$ and $q=\frac{1}{2}n$. The four-dimensional reps appear only for $n\geqslant 3$. In the first column the short rep symbols are defined.

Rep	$(C_n^s\|t)$	$(\sigma_v C_n^s\|t)$	$(U_d C_n^s\|-t)$	$(U_d\sigma_v C_n^s\|-t)$
$(0A0\pm)={}_0A_0^{\pm}$	1	1	±1	±1
$(0B0\pm)={}_0B_0^{\pm}$	1	-1	±1	∓1
$(0Em\pm)={}_0E_{m,-m}^{\pm}$	$2\cos(ms\alpha)$	0	0	$\pm2\cos(m(s+\frac{1}{2})\alpha)$
$(kEA0)={}_k^{-k}E_{A_0}$	$2\cos(kt)$	$2\cos(kt)$	0	0
$(kEB0)={}_k^{-k}E_{B_0}$	$2\cos(kt)$	$-2\cos(kt)$	0	0
$(kGm)={}_k^{-k}G_{m,-m}$	$4\cos(ms\alpha)\cos(kt)$	0	0	0

***Table* 1.** *(continued)*

Rep	$(C_n^s\|t)$	$(\sigma_v C_n^s\|t)$	$(U_d C_n^s\|-t)$	$(U_d \sigma_v C_n^s\|-t)$
$(\pi Ao\pm) = {}_\pi A_0^\pm$	$(-1)^t$	$(-1)^t$	$\pm(-1)^t$	$\pm(-1)^t$
$(\pi Bo\pm) = {}_\pi B_0^\pm$	$(-1)^t$	$-(-1)^t$	$\pm(-1)^t$	$\mp(-1)^t$
$(\pi Em\pm) = {}_\pi E^\pm_{m,-m}$	$2(-1)^t \cos(ms\alpha)$	0	0	$\pm 2(-1)^t \cos(m(s+\frac{1}{2})\alpha)$
and only for $n = 2q = 2, 4, 6, \ldots$				
$(0Eq) = {}_0E_q$	$2(-1)^s$	0	0	0
$(kEAq) = {}^{-k}_{k}E^{B_q}_{A_q}$	$2(-1)^s \cos(kt)$	$2\mathrm{i}(-1)^s \sin(kt)$	0	0
$(kEB_q) = {}^{-k}_{k}E^{A_q}_{B_q}$	$2(-1)^s \cos(kt)$	$-2\mathrm{i}(-1)^s \sin(kt)$	0	0
$(\pi Eq) = {}_\pi E_q$	$2(-1)^{s+t}$	0	0	0

Table 2. Decompositions of the Kronecker products of reps of **L$\bar{n}$m** (n odd) and **L$(\overline{2n})$2m** (n even). The rep symbols are defined in table 1.

(0*A*o+) × (0*A*o+) = (0*A*o+)

(0*A*o−) × (0*A*o±) = (0*A*o∓)

(0*B*o+) × (0*A*o±) = (0*B*o±) (0*B*o+) × (0*B*o+) = (0*A*o+)

(0*B*o−) × (0*A*o±) = (0*B*o∓) (0*B*o−) × (0*B*o±) = (0*A*o∓)

(0*Em*+) × (0*A*o±) = (0*Em*+) × (0*B*o∓) = (0*Em*±)
(0*Em*+) × (0*Em'*+) = (1) (0*Ev*+) + (0*Ew*+) (2) (0*A*o+) + (0*B*o−) + (0*Ew*+)
(3) (0*Ev*+) + (0*Eq*) (4) (0*A*o+) + (0*B*o−) + (0*Eq*)

(0*Em*−) × (0*A*o±) = (0*Em*−) × (0*B*o∓) = (0*Em*∓)
(0*Em*−) × (0*Em'*±) = (1) (0*Ev*∓) + (0*Ew*∓) (2) (0*A*o∓) + (0*B*o±) + (0*Ew*∓)
(3) (0*Ev*∓) + (0*Eq*) (4) (0*A*o∓) + (0*B*o±) + (0*Eq*)

(*kEA*o) × (0*A*o±) = (*kEA*o) (*kEA*o) × (0*B*o±) = (*kEB*o) (*kEA*o) × (0*Em*±) = (*kGm*)
(*kEA*o) × (*k'EA*o) = (a) (*xEA*o) + (*yEA*o) (b) (0*A*o+) + (0*A*o−) + (*yEA*o)
(c) (*xEA*o) + (π*A*o+) + (π*A*o−) (d) (0*A*o+) + (0*A*o−) + (π*A*o+) + (π*A*o−)

(*kEB*o) × (0*A*o±) = (*kEB*o) (*kEB*o) × (0*B*o±) = (*kEA*o) (*kEB*o) × (0*Em*±) = (*kGm*)
(*kEB*o) × (*k'EA*o) = (a) (*xEB*o) + (*yEB*o) (b) (0*B*o+) + (0*B*o−) + (*yEB*o)
(c) (*xEB*o) + (π*B*o+) + (π*B*o−) (d) (0*B*o+) + (0*B*o−) + (π*B*o+) + (π*B*o−)
(*kEB*o) × (*k'EB*o) = (a) (*xEA*o) + (*yEA*o) (b) (0*A*o+) + (0*A*o−) + (*yEA*o)
(c) (*xEA*o) + (π*A*o+) + (π*A*o−) (d) (0*A*o+) + (0*A*o−) + (π*A*o+) + (π*A*o−)

(*kGm*) × (0*A*o±) = (*kGm*) × (0*B*o±) = (*kGm*)
(*kGm*) × (0*Em'*±) = (1) (*kGv*) + (*kGw*) (2) (*kEA*o) + (*kEB*o) + (*kGw*)
(3) (*kGv*) + (*kEAq*) + (*kEBq*) (4) (*kEA*o) + (*kEB*o) + (*kEAq*) + (*kEBq*)
(*kGm*) × (*k'EA*o) = (*kGm*) × (*k'EB*o) =
(a) (*xGm*) + (*yGm*) (b) (0*Em*+) + (0*Em*−) + (*yGm*)
(c) (*xGm*) + (π*Em*+) + (π*Em*−) (d) (0*Em*+) + (0*Em*−) + (π*Em*+) + (π*Em*−)
(*kGm*) × (*k'Gm'*) = (a1) (*xGv*) + (*xGw*) + (*yGv*) + (*yGw*)
(a2) (*xEA*o) + (*xEB*o) + (*xGw*) + (*yEA*o) + (*yEB*o) + (*yGw*)
(a3) (*xGv*) + (*xEAq*) + (*xEBq*) + (*yGv*) + (*yEAq*) + (*yEBq*)
(a4) (*xEA*o) + (*xEB*o) + (*xEAq*) + (*xEBq*) + (*yEA*o) + (*yEB*o) + (*yEAq*) + (*yEBq*)
(b1) (0*Ev*+) + (0*Ev*−) + (0*Ew*+) + (0*Ew*−) + (*yGv*) + (*yGw*)
(b2) (0*A*o+) + (0*A*o−) + (0*B*o+) + (0*B*o−) + (0*Ew*+) + (0*Ew*−) + (*yEA*o) + (*yEB*o) + (*yGw*)
(b3) (0*Ev*+) + (0*Ev*−) + 2(0*Eq*) + (*yGv*) + (*yEAq*) + (*yEBq*)
(b4) (0*A*o+) + (0*A*o−) + (0*B*o+) + (0*B*o−) + 2(0*Eq*) + (*yEA*o) + (*yEB*o) + (*yEAq*) + (*yEBq*)
(c1) (*xGv*) + (*xGw*) + (π*Ev*+) + (π*Ev*−) + (π*Ew*+) + (π*Ew*−)
(c2) (*xEA*o) + (*xEB*o) + (*xGw*) + (π*A*o+) + (π*A*o−) + (π*B*o+) + (π*B*o−) + (π*Ew*+) + (π*Ew*−)

Table 2. (continued)

(c3) $(xGv)+(xEAq)+(xEBq)+(\pi Ev+)+(\pi Ev-)+2(\pi Eq)$

(c4) $(xE A\mathrm{o})+(xEB\mathrm{o})+(xEAq)+(xEBq)+(\pi A\mathrm{o}+)+(\pi B\mathrm{o}-)+(\pi A\mathrm{o}-)+(\pi B\mathrm{o}+)+2(\pi Eq)$

(d1) $(\mathrm{o}Ev+)+(\mathrm{o}Ev-)+(\mathrm{o}Ew+)+(\mathrm{o}Ew-)+(\pi Ev+)+(\pi Ev-)+(\pi Ew+)+(\pi Ew-)$

(d2) $(\mathrm{o}A\mathrm{o}+)+(\mathrm{o}A\mathrm{o}-)+(\mathrm{o}B\mathrm{o}+)+(\mathrm{o}B\mathrm{o}-)+(\mathrm{o}Ew+)+(\mathrm{o}Ew-)+(\pi A\mathrm{o}+)+(\pi A\mathrm{o}-)$
$+(\pi B\mathrm{o}+)+(\pi B\mathrm{o}-)+(\pi Ew+)+(\pi Ew-)$

(d3) $(\mathrm{o}Ev+)+(\mathrm{o}Ev-)+2(\mathrm{o}Eq)+(\pi Ev+)+(\pi Ev-)+2(\pi Eq)$

(d4) $(\mathrm{o}A\mathrm{o}+)+(\mathrm{o}A\mathrm{o}-)+(\mathrm{o}B\mathrm{o}+)+(\mathrm{o}B\mathrm{o}-)+2(\mathrm{o}Eq)+(\pi A\mathrm{o}+)+(\pi B\mathrm{o}-)$
$+(\pi A\mathrm{o}-)+(\pi B\mathrm{o}+)+2(\pi Eq)$

$(\boldsymbol{\pi A\mathrm{o}+})\times(\mathrm{o}A\mathrm{o}\pm)=(\pi A\mathrm{o}\pm)$ $(\pi A\mathrm{o}+)\times(\mathrm{o}B\mathrm{o}\pm)=(\pi B\mathrm{o}\pm)$ $(\pi A\mathrm{o}+)\times(\mathrm{o}Em\pm)=(\pi Em\pm)$
$(\pi A\mathrm{o}+)\times(kEA\mathrm{o})=(zEA\mathrm{o})$ $(\pi A\mathrm{o}+)\times(kEB\mathrm{o})=(zEB\mathrm{o})$ $(\pi A\mathrm{o}+)\times(kGm)=(zGm)$
$(\pi A\mathrm{o}+)\times(\pi A\mathrm{o}+)=(\mathrm{o}A\mathrm{o}+)$

$(\boldsymbol{\pi A\mathrm{o}-})\times(\mathrm{o}A\mathrm{o}\pm)=(\pi A\mathrm{o}\mp)$ $(\pi A\mathrm{o}-)\times(\mathrm{o}B\mathrm{o}\pm)=(\pi B\mathrm{o}\mp)$ $(\pi A\mathrm{o}-)\times(\mathrm{o}Em\pm)=(\pi Em\mp)$
$(\pi A\mathrm{o}-)\times(kEA\mathrm{o})=(zEA\mathrm{o})$ $(\pi A\mathrm{o}-)\times(kEB\mathrm{o})=(zEB\mathrm{o})$ $(\pi A\mathrm{o}-)\times(kGm)=(zGm)$
$(\pi A\mathrm{o}-)\times(\pi A\mathrm{o}\pm)=(\mathrm{o}A\mathrm{o}\mp)$

$(\boldsymbol{\pi B\mathrm{o}+})\times(\mathrm{o}A\mathrm{o}\pm)=(\pi B\mathrm{o}\mp)$ $(\pi B\mathrm{o}+)\times(\mathrm{o}B\mathrm{o}\pm)=(\pi A\mathrm{o}\pm)$ $(\pi B\mathrm{o}+)\times(\mathrm{o}Em\pm)=(\pi Em\mp)$
$(\pi B\mathrm{o}+)\times(kEA\mathrm{o})=(zEB\mathrm{o})$ $(\pi B\mathrm{o}+)\times(kEB\mathrm{o})=(zEA\mathrm{o})$ $(\pi B\mathrm{o}+)\times(kGm)=(zGm)$
$(\pi B\mathrm{o}+)\times(\pi A\mathrm{o}\pm)=(\mathrm{o}B\mathrm{o}\pm)$ $(\pi B\mathrm{o}+)\times(\pi B\mathrm{o}+)=(\mathrm{o}A\mathrm{o}+)$

$(\boldsymbol{\pi B\mathrm{o}-})\times(\mathrm{o}A\mathrm{o}\pm)=(\pi B\mathrm{o}\mp)$ $(\pi B\mathrm{o}-)\times(\mathrm{o}B\mathrm{o}\pm)=(\pi A\mathrm{o}\mp)$ $(\pi B\mathrm{o}-)\times(\mathrm{o}Em\pm)=(\pi Em\pm)$
$(\pi B\mathrm{o}-)\times(kEA\mathrm{o})=(zEB\mathrm{o})$ $(\pi B\mathrm{o}-)\times(kEB\mathrm{o})=(zEA_0)$ $(\pi B\mathrm{o}-)\times(kGm)=(zGm)$
$(\pi B\mathrm{o}-)\times(\pi A\mathrm{o}\pm)=(\mathrm{o}B\mathrm{o}\mp)$ $(\pi B\mathrm{o}-)\times(\pi B\mathrm{o}\pm)=(\mathrm{o}A\mathrm{o}\mp)$

$(\boldsymbol{\pi Em+})\times(\mathrm{o}A\mathrm{o}\pm)=(\pi Em+)\times(\mathrm{o}B\mathrm{o}\mp)=(\pi Em+)$
$(\pi Em+)\times(\mathrm{o}Em'\pm)=$ (1) $(\pi Ev\pm)+(\pi Ew\pm)$ (2) $(\pi A\mathrm{o}\pm)+(\pi B\mathrm{o}\mp)+(\pi Ew\pm)$
(3) $(\pi Ev\pm)+(\pi Eq)$ (4) $(\pi A\mathrm{o}\pm)+(\pi B\mathrm{o}\mp)+(\pi Eq)$
$(\pi Em+)=(kEA\mathrm{o})=(\pi Em+)\times(kEB\mathrm{o})=(zGm)$
$(\pi Em+)\times(kGm')=$ (1) $(zGv)+(zGw)$ (2) $(zEA\mathrm{o})+(zEB\mathrm{o})+(zGw)$
(3) $(zGv)+(zEAq)+(zEBq)$ (4) $(zEA\mathrm{o})+(zEAq)+(zEAq)+(zEBq)$
$(\pi Em+)\times(\pi A\mathrm{o}\pm)=(\pi Em+)\times(\pi B\mathrm{o}\mp)=(\mathrm{o}Em\pm)$
$(\pi Em+)\times(\pi Em'+)=$ (1) $(\mathrm{o}Ev+)+(\mathrm{o}Ew+)$ (2) $(\mathrm{o}A\mathrm{o}+)+(\mathrm{o}B\mathrm{o}-)+(\mathrm{o}Ew+)$
(3) $(\mathrm{o}Ev+)+(\mathrm{o}Eq)$ (4) $(\mathrm{o}A\mathrm{o}+)+(\mathrm{o}B\mathrm{o}-)+(\mathrm{o}Eq)$

$(\boldsymbol{\pi Em-})\times(\mathrm{o}A\mathrm{o}\pm)=(\pi Em-)\times(\mathrm{o}B\mathrm{o}\mp)=(\pi Em\mp)$
$(\pi Em-)\times(\mathrm{o}Em'\pm)=$ (1) $(\pi Ev\mp)+(\pi Ew\mp)$ (2) $(\pi A\mathrm{o}\mp)+(\pi B\mathrm{o}\pm)+(\pi Ew\mp)$
(3) $(\pi Ev\mp)+(\pi Eq)$ (4) $(\pi A\mathrm{o}\mp)+(\pi B\mathrm{o}\pm)+(\pi Eq)$
$(\pi Em-)\times(kEA\mathrm{o})=(\pi Em-)\times(kEB\mathrm{o})=(zGm)$
$(\pi Em-)\times(kGm')=$ (1) $(zGv)+(zGw)$ (2) $(zEA\mathrm{o})+(zEB\mathrm{o})+(zGw)$
(3) $(zGv)+(zEAq)+(zEBq)$ (4) $(zEA\mathrm{o})+(zEB\mathrm{o})+(zEAq)+(zEBq)$
$(\pi Em-)\times(\pi A\mathrm{o}\pm)=(\pi Em-)\times(\pi B\mathrm{o}\mp)=(\mathrm{o}Em\mp)$
$(\pi Em-)\times(\pi Em'\pm)=$ (1) $(\mathrm{o}Ev\mp)+(\mathrm{o}Ew\mp)$ (2) $(\mathrm{o}A\mathrm{o}\mp)+(\mathrm{o}B\mathrm{o}\pm)+(\mathrm{o}Ew\mp)$
(3) $(\mathrm{o}Ev\mp)+(\mathrm{o}Eq)$ (4) $(\mathrm{o}A\mathrm{o}\mp)+(\mathrm{o}B\mathrm{o}\pm)+(\mathrm{o}Eq)$

and only for $n=2q=2, 4, 6, \ldots$

$(\boldsymbol{\mathrm{o}Eq})\times(\mathrm{o}A\mathrm{o}\pm)=(\mathrm{o}Eq)\times(\mathrm{o}B\mathrm{o}\pm)=(\mathrm{o}Eq)$ $(\mathrm{o}Eq)\times(\mathrm{o}Em\pm)=(\mathrm{o}Eu+)+(\mathrm{o}Eu-)$
$(\mathrm{o}Eq)\times(kEA\mathrm{o})=(\mathrm{o}Eq)\times(kEB\mathrm{o})=(kEAq)+(kEBq)$ $(\mathrm{o}Eq)\times(kGm)=2(kGu)$
$(\mathrm{o}Eq)\times(\pi A\mathrm{o}\pm)=(\mathrm{o}Eq)\times(\pi B\mathrm{o}\pm)=(\pi Eq)$ $(\mathrm{o}Eq)\times(\pi Em\pm)=(\pi Eu+)+(\pi Eu-)$
$(\mathrm{o}Eq)\times(\mathrm{o}Eq)=(\mathrm{o}A\mathrm{o}+)+(\mathrm{o}A\mathrm{o}-)+(\mathrm{o}B\mathrm{o}+)+(\mathrm{o}B\mathrm{o}-)$

$(\boldsymbol{kEAq})\times(\mathrm{o}A\mathrm{o}\pm)=(kEAq)$ $(kEAq)\times(\mathrm{o}B\mathrm{o}\pm)=(kEBq)$ $(kEAq)\times(\mathrm{o}Em\pm)=(kGu)$

$(kEAq)\times(k'EA\mathrm{o})=$ (a) $(xEAq)+(yEAq)$ (b) $(\mathrm{o}Eq)+(yEAq)$
(c) $(xEAq)+(\pi Eq)$ (d) $(\mathrm{o}Eq)+(\pi Eq)$
$(kEAq)\times(k'EB\mathrm{o})=$ (a) $(xEBq)+(yEBq)$ (b) $(\mathrm{o}Eq)+(yEBq)$
(c) $(xEBq)+(\pi Eq)$ (d) $(\mathrm{o}Eq)+(\pi Eq)$

Table 2. (continued)

$(kEAq)\times(k'Gm)=$ (a) $(xGu)+(yGu)$ (b) $(0Eu+)+(0Eu-)+(yGu)$
(c) $(xGu)+(\pi Eu+)+(\pi Eu-)$ (d) $(0Eu+)+(0Eu-)+(\pi Eu+)+(\pi Eu-)$
$(kEAq)\times(\pi A0\pm)=(zEBq)$ $(kEAq)\times(\pi B0\pm)=(zEAq)$ $(kEAq)\times(\pi Em\pm)=(zGu)$
$(kEAq)\times(0Eq)=(kEA0)+(kEB0)$
$(kEAq)\times(k'EAq)=$ (a) $(xEB0)+(yEA0)$ (b) $(0B0+)+(0B0-)+(yEA0)$
(c) $(xEB0)+(\pi A0+)+(\pi A0-)$ (d) $(0B0+)+(0B0-)+(\pi A0+)+(\pi A0-)$

(kEBq) $\times(0A0\pm)=(kEBq)$ $(kEBq)\times(0B0\pm)=(kEAq)$ $(kEBq)\times(0Em\pm)=(kGu)$
$(kEBq)=(k'EA0)=$ (a) $(xEBq)+(yEBq)$ (b) $(0Eq)+(yEBq)$
(c) $(xEBq)+(\pi Eq)$ (d) $(0Eq)+(\pi Eq)$
$(kEBq)\times(k'EB0)=$ (a) $(xEAq)+(yEAq)$ (b) $(0Eq)+(yEAq)$
(c) $(xEAq)+(\pi Eq)$ (d) $(0Eq)+(\pi Eq)$
$(kEBq)\times(k'Gm)=$ (a) $(xGu)+(yGu)$ (b) $(0Eu+)+(0Eu-)+(yGu)$
(c) $(xGu)+(\pi Eu+)+(\pi Eu-)$ (d) $(0Eu+)+(0Eu-)+(\pi Eu+)+(\pi Eu-)$
$(kEBq)\times(\pi A0\pm)=(zEAq)$ $(kEBq)\times(\pi B0\pm)=(zEBq)$ $(kEBq)\times(\pi Em\pm)=(zGu)$
$(kEBq)\times(0Eq)=(kEA0)+(kEB0)$
$(kEBq)\times(k'EAq)=$ (a) $(xEA0)+(yEB0)$ (b) $(0A0+)+(0A0-)+(yEB0)$
(c) $(xEA0)+(\pi B0+)+(\pi B0-)$ (d) $(0A0+)+(0A0-)+(\pi B0+)+(\pi B0-)$
$(kEBq)\times(k'EBq)=$ (a) $(xEB0)+(yEA0)$ (b) $(0B0+)+(0B0-)+(yEA0)$
(c) $(xEB0)+(\pi A0+)+(\pi A0-)$ (d) $(0B0+)+(0B0-)+(\pi A0+)+(\pi A0-)$

(πEq) $\times(0A0\pm)=(\pi Eq)\times(0B0\pm)=(\pi Eq)$ $(\pi Eq)\times(0Em\pm)=(\pi Eu+)+(\pi Eu-)$
$(\pi Eq)\times(kEA0)=(\pi Eq)\times(kEB0)=(zEAq)+(zEBq)$ $(\pi Eq)\times(kGm)=2(zGu)$
$(\pi Eq)\times(\pi A0\pm)=(\pi Eq)\times(\pi B0\pm)=(0Eq)$ $(\pi Eq)\times(\pi Em\pm)=(0Eu+)+(0Eu-)$
$(\pi Eq)\times(0Eq)=(\pi A0+)+(\pi A0-)+(\pi B0+)+(\pi B0-)$
$(\pi Eq)\times(kEAq)=(\pi Eq)\times(kEBq)=(zEA0)+(zEB0)$
$(\pi Eq)\times(\pi Eq)=(0A0+)+(0A0-)+(0B0+)+(0B0-)$

3.2. The non-symmorphic line groups **L$\bar{n}$c** *($n = 1, 3, 5, \ldots$) and* **L$(\overline{2n})$2c** *($n = 2, 4, 6, \ldots$).*

Table 3. The characters of the reps of the line groups **L$\bar{n}$c** ($n = 1, 3, 5, \ldots$) and **L$(\overline{2n})$2c** ($n = 2, 4, 6, \ldots$). For s, t, α, k, m and q see the caption of table 1.

Rep	$(C_n^s\|t)$	$(\sigma_v C_n^s\|t+\frac{1}{2})$	$(U_d C_n^s\|-t)$	$(U_d\sigma_v C_n^s\|-t-\frac{1}{2})$
$(0A0\pm)={}_0A_0^{\pm}$	1	1	± 1	± 1
$(0B0\pm)={}_0B_0^{\pm}$	1	-1	± 1	∓ 1
$(0Em\pm)={}_0E_{m,-m}^{\pm}$	$2\cos(ms\alpha)$	0	0	$\pm 2\cos(m(s+\frac{1}{2})\alpha)$
$(kEA0)={}^{-k}_{k}E_{A_0}$	$2\cos(kt)$	$2\cos(k(t+\frac{1}{2}))$	0	0
$(kEB0)={}^{-k}_{k}E_{B_0}$	$2\cos(kt)$	$-2\cos(k(t+\frac{1}{2}))$	0	0
$(kGm)={}^{-k}_{k}G_{m,-m}$	$4\cos(kt)\cos(ms\alpha)$	0	0	0
$(\pi E0)={}_{\pi}E_0$	$2(-1)^t$	0	0	0
$(\pi Em\pm)={}_{\pi}E_{m,-m}^{\pm}$	$2(-1)^t\cos(ms\alpha)$	0	0	$\mp 2\mathrm{i}(-1)^t\sin(m(s+\frac{1}{2})\alpha)$
and only for $n=2q=2, 4, 6, \ldots$				
$(0Eq)={}_0E_q$	$2(-1)^s$	0	0	0
$(kEAq)={}^{-k}_{k}E_{A_q}^{B_q}$	$2(-1)^s\cos(kt)$	$2\mathrm{i}(-1)^s\sin(k(t+\frac{1}{2}))$	0	0
$(kEBq)={}^{-k}_{k}E_{B_q}^{A_q}$	$2(-1)^s\cos(kt)$	$-2\mathrm{i}(-1)^s\sin(k(t+\frac{1}{2}))$	0	0
$(\pi Aq\pm)={}_{\pi}A_q^{\pm}$	$(-1)^{s+t}$	$\mathrm{i}(-1)^{s+t}$	$\pm(-1)^{s+t}$	$\pm\mathrm{i}(-1)^{s+t}$
$(\pi Bq\pm)={}_{\pi}B_q^{\pm}$	$(-1)^{s+t}$	$-\mathrm{i}(-1)^{s+t}$	$\pm(-1)^{s+t})$	$\mp\mathrm{i}(-1)^{s+t}$

Table 4. Decompositions of the Kronecker products of reps of $\mathbf{L}\bar{n}c$ (n odd) and $\mathbf{L}(\overline{2n})2c$ (n even). The rep symbols are defined in table 3.

$\mathbf{(0A0+)}\times(0A0+)=(0A0+)$

$\mathbf{(0A0-)}\times(0A0\pm)=(0A0\mp)$

$\mathbf{(0B0+)}\times(0A0\pm)=(0B0\pm)$ $\quad (0B0+)\times(0B0+)=(0A0+)$

$\mathbf{(0B0-)}\times(0A0\pm)=(0B0\mp)$ $\quad (0B0-)\times(0B0\pm)=(0A0\mp)$

$\mathbf{(0Em+)}\times(0A0\pm)=(0Em+)\times(0B0\mp)=(0Em\pm)$
$(0Em+)\times(0Em'+)=$ (1) $(0Ev+)+(0Ew+)$ $\quad$ (2) $(0A0+)+(0B0-)+(0Ew+)$
(3) $(0Ev+)+(0Eq)$ $\quad$ (4) $(0A0+)+(0B0-)+(0Eq)$

$\mathbf{(0Em-)}\times(0A0\pm)=(0Em-)\times(0B0\mp)=(0Em\mp)$
$(0Em-)\times(0Em'\pm)=$ (1) $(0Ev\mp)+(0Ew\mp)$ $\quad$ (2) $(0A0\mp)+(0B0\pm)+(0Ew\mp)$
(3) $(0Ev\mp)+(0Eq)$ $\quad$ (4) $(0A0\mp)+(0B0\pm)+(0Eq)$

$\mathbf{(kEA0)}\times(0A0\pm)=(kEA0)$ $\quad (kEA0)\times(0B0\pm)=(kEB0)$ $\quad (kEA0)\times(0Em\pm)=(kGm)$
$(kEA0)\times(k'EA0)=$ (e) $(xEA0)+(yEA0)$ $\quad$ (f) $(xEA0)+(yEB0)$
(g) $(0A0+)+(0A0-)+(yEA0)$ $\quad$ (h) $(0A0+)+(0A0-)+(yEB0)$
(c) $(xEA0)+(\pi E0)$ $\quad$ (d) $(0A0+)+(0A0-)+(\pi E0)$

$\mathbf{(kEB0)}\times(0A0\pm)=(kEB0)$ $\quad (kEB0)\times(0B0\pm)=(kEA0)$ $\quad (kEB0)\times(0Em\pm)=(kGm)$
$(kEB0)\times(k'EA0)=$ (e) $(xEB0)+(yEB0)$ $\quad$ (f) $(xEB0)+(yEA0)$
(g) $(0B0+)+(0B0-)+(yEB0)$ $\quad$ (h) $(0B0+)+(0B0-)+(yEA0)$
(c) $(xEB0)+(\pi E0)$ $\quad$ (d) $(0B0+)+(0B0-)+(\pi E0)$
$(kEB0)\times(k'EB0)=$ (e) $(xEA0)+(yEA0)$ $\quad$ (f) $(xEA0)+(yEB0)$
(g) $(0A0+)+(0A0-)+(yEA0)$ $\quad$ (h) $(0A0+)+(0A0-)+(yEB0)$
(c) $(xEA0)+(\pi E0)$ $\quad$ (d) $(0A0+)+(0A0-)+(\pi E0)$

$\mathbf{(kGm)}\times(0A0\pm)=(kGm)\times(0B0\pm)=(kGm)$
$(kGm)\times(0Em'\pm)=$ (1) $(kGv)+(kGw)$ $\quad$ (2) $(kEA0)+(kEB0)+(kGw)$
(3) $(kGv)+(kEAq)+(kEBq)$ $\quad$ (4) $(kEA0)+(kEB0)+(kEAq)+(kEBq)$
$(kGm)\times(k'EA0)=(kGm)\times(k'EB0)=$
(1) $(xGm)\times(yGm)$ $\quad$ (2) $(0Em+)+(0Em-)+(yGm)$
(3) $(xGm)+(\pi Em+)+(\pi Em-)$ $\quad$ (4) $(0Em+)+(0Em-)+(\pi Em+)+(\pi Em-)$
$(kGm)\times(k'Gm')=$
(a1) $(xGv)+(xGw)+(yGv)+(yGw)$
(a2) $(xEA0)+(xEB0)+(xGw)+(yEA0)+(yEB0)+(yGw)$
(a3) $(xGv)+(xEAq)+(xEBq)+(yGv)+(yEAq)+(yEBq)$
(a4) $(xEA0)+(xEB0)+(xEAq)+(xEBq)+(yEA0)+(yEB0)+(yEAq)+(yEBq)$
(b1) $(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(yGv)+(yGw)$
(b2) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+(yEA0)+(yEB0)+(yGw)$
(b3) $(0Ev+)+(0Ev-)+2(0Eq)+(yGv)+(yEAq)+(yEBq)$
(b4) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+2(0Eq)+(yEA0)+(yEB0)+(yEAq)+(yEBq)$
(c1) $(xGv)+(xGw)+(\pi Ev+)+(\pi Ev-)+(\pi Ew+)+(\pi Ew-)$
(c2) $(xEA0)+(xEB0)+(xGw)+2(\pi E0)+(\pi Ew+)+(\pi Ew-)$
(c3) $(xGv)+(xEAq)+(xEBq)+(\pi Ev+)+(\pi Ev-)+(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)$
(c4) $(xEA0)+(xEB0)+(xEAq)+(xEBq)+2(\pi E0)+(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)$
(d1) $(0Ev+)+(0Ev-)+(0Ew+)+(0Ew-)+(\pi Ev+)+(\pi Ev-)+(\pi Ew+)+(\pi Ew-)$
(d2) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+(0Ew+)+(0Ew-)+2(\pi E0)+(\pi Ew+)+(\pi Ew-)$
(d3) $(0Ev+)+(0Ev-)+2(0Eq)+(\pi Ev+)+(\pi Ev-)+(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)$
(d4) $(0A0+)+(0A0-)+(0B0+)+(0B0-)+2(0Eq+)+2(\pi E0)+(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)$

$\mathbf{(\pi E0)}\times(0A0\pm)=(\pi E0)\times(0B0\pm)=(\pi E0)$ $\quad (\pi E0)\times(0Em\pm)=(\pi Em+)+(\pi Em-)$
$(\pi E0)\times(kEA0)=(\pi E0)\times(kEB0)=(zEA0)+(zEB0)$ $\quad (\pi E0)\times(kGm)=2(zGm)$
$(\pi E0)\times(\pi E0)=(0A0+)+(0A0-)+(0B0+)+(0B0-)$

$\mathbf{(\pi Em+)}\times(0A0\pm)=(\pi Em+)\times(0B0\mp)=(\pi Em\pm)$
$(\pi Em+)\times(0Em'\pm)=$ (1) $(\pi Ev\pm)+(\pi Ew\pm)$ $\quad$ (2) $(\pi E0)+(\pi Ew\pm)$
(3) $(\pi Ev\pm)+(\pi Aq\mp)+(\pi Bq\pm)$ $\quad$ (4) $(\pi E0)+(\pi Aq\mp)+(\pi Bq\pm)$

Table 4. (continued)

$(\pi Em+)\times(kEA0)=(\pi Em+)\times(kEB0)=(zGw)$

$(\pi Em+)\times(kGm')=(1)\ (zGv)+(zGw)\qquad(2)\ (zEA0)+(zEB0)+(zGw)$

$(3)\ (zGv)+(zEAq)+(zEBq)\qquad(4)\ (zEA0)+(zEB0)+(zEAq)+(zEBq)$

$(\pi Em+)\times(\pi E0)=(0Em+)+(0Em-)$

$(\pi Em+)\times(\pi Em'+)=(1)\ (0Ev-)+(0Ew+)\qquad(2)\ (0A0-)+(0B0+)+(0Ew+)$

$(3)\ (0Ev-)+(0Eq)\qquad(4)\ (0A0-)+(0B0+)+(0Eq)$

$\mathbf{(\pi Em-)}\times(0A0\pm)=(\pi Em-)\times(0B0\mp)=(\pi Em\mp)$

$(\pi Em-)\times(0Em'\pm)=(1)\ (\pi Ev\mp)+(\pi Ew\mp)\qquad(2)\ (\pi E0)+(\pi Ew\mp)$

$(3)\ (\pi Ev\mp)+(\pi Aq\pm)+(\pi Bq\mp)\qquad(4)\ (\pi E0)+(\pi Aq\pm)+(\pi Bq\mp)$

$(\pi Em-)\times(kEA0)=(\pi Em-)\times(kEB0)=(zGm)$

$(\pi Em-)\times(kGm')=(1)\ (zGv)+(zGw)\qquad(2)\ (zEA0)+(zEB0)+(zGw)$

$(3)\ (zGv)+(zEAq)+(zEBq)\qquad(4)\ (zEA0)+(zEB0)+(zEAq)+(zEBq)$

$(\pi Em-)\times(\pi E0)=(0Em+)+(0Em-)$

$(\pi Em-)\times(\pi Em'\pm)=(1)\ (0Ev\pm)+(0Ew\mp)\qquad(2)\ (0A0\pm)+(0B0\mp)+(0Ew\mp)$

$(3)\ (0Ev\pm)+(0Eq)\qquad(4)\ (0A0\pm)+(0B0\mp)+(0Eq)$

and only for $n=2q=2, 4, 6, \ldots$

$\mathbf{(0Eq)}\times(0A0\pm)=(0Eq)\times(0B0\pm)=(0Eq)\qquad(0Eq)\times(0Em\pm)=(0Eu+)+(0Eu-)$

$(0Eq)\times(kEA0)=(0Eq)\times(kEB0)=(kEAq)+(kEBq)\qquad(0Eq)\times(kGm)=2(kGu)$

$(0Eq)\times(\pi E0)=(\pi Aq+)+(\pi Aq-)+(\pi Bq+)+(\pi Bq-)\qquad(0Eq)\times(\pi Em\pm)=(\pi Eu+)+(\pi Eu-)$

$(0Eq)\times(0Eq)=(0A0+)+(0A0-)+(0B0+)+(0B0-)$

$\mathbf{(kEAq)}\times(0A0\pm)=(kEAq)\qquad(kEAq)\times(0B0\pm)=(kEBq)\qquad(kEAq)\times(0Em\pm)=(kGu)$

$(kEAq)\times(k'EA0)=(a)\ (xEAq)+(yEAq)\qquad(b)\ (0Eq)+(yEAq)$

$(c)\ (xEAq)+(\pi Aq+)+(\pi Aq-)\qquad(d)\ (0Eq)+(\pi Aq+)+(\pi Aq-)$

$(kEAq)\times(k'EB0)=(a)\ (xEBq)+(yEBq)\qquad(b)\ (0Eq)+(yEBq)$

$(c)\ (xEBq)+(\pi Bq+)+(\pi Bq-)\qquad(d)\ (0Eq)+(\pi Bq+)+(\pi Bq-)$

$(kEAq)\times(k'Gm)=(a)\ (xGu)+(yGu)\qquad(b)\ (0Eu+)+(0Eu-)+(yGu)$

$(c)\ (xGu)+(\pi Eu+)+(\pi Eu-)\qquad(d)\ (0Eu+)+(0Eu-)+(\pi Eu+)+(\pi Eu-)$

$(kEAq)\times(\pi E0)=(zEAq)+(zEBq)\qquad(kEAq)\times(\pi Em\pm)=(zGu)$

$(kEAq)\times(0Eq)=(kEA0)+(kEB0)$

$(kEAq)\times(k'EAq)=(e)\ (xEB0)+(yEA0)\qquad(f)\ (xEB0)+(yEB0)$

$(g)\ (0B0+)+(0B0-)+(yEA0)\qquad(h)\ (0B0+)+(0B0-)+(yEB0)$

$(c)\ (xEB0)+(\pi E0)\qquad(d)\ (0B0+)+(0B0-)+(\pi E0)$

$\mathbf{(kEBq)}\times(0A0\pm)=(kEBq)\qquad(kEBq)\times(0B0\pm)=(kEAq)\qquad(kEBq)\times(0Em\pm)=(kGh)$

$(kEBq)\times(k'EA0)=(a)\ (xEBq)+(yEBq)\qquad(b)\ (0Eq)+(yEBq)$

$(c)\ (xEBq)+(\pi Bq+)+(\pi Bq-)\qquad(d)\ (0Eq)+(\pi Bq+)+(\pi Bq-)$

$(kEBq)\times(k'EB0)=(a)\ (xEAq)+(yEAq)\qquad(b)\ (0Eq)+(yEAq)$

$(c)\ (xEAq)+(\pi Aq+)+(\pi Aq-)\qquad(d)\ (0Eq)+(\pi Aq+)+(\pi Aq-)$

$(kEBq)\times(k'Gm)=(a)\ (xGu)+(yGu)\qquad(b)\ (0Eu+)+(0Eu-)+(yGu)$

$(c)\ (xGu)+(\pi Eu+)+(\pi Eu-)\qquad(d)\ (0Eu+)+(0Eu+)+(\pi Eu-)+(\pi Eu-)$

$(kEBq)\times(\pi E0)=(zEAq)+(zEBq)\qquad(kEBq)\times(\pi Em\pm)=(zGu)$

$(kEBq)\times(0Eq)=(kEA0)+(kEB0)$

$(kEBq)\times(k'EAq)=(e)\ (xEA0)+(yEA0)\qquad(f)\ (xEA0)+(yEA0)$

$(g)\ (0A0+)+(0A0-)+(yEB0)\qquad(h)\ (0A0+)+(0A0-)+(yEA0)$

$(c)\ (xEA0)+(\pi E0)\qquad(d)\ (0A0+)+(0A0-)+(\pi E0)$

$(kEBq)\times(k'EBq)=(e)\ (xEB0)+(yEA0)\qquad(f)\ (xEB0)+(yEB0)$

$(g)\ (0B0+)+(0B0-)+(yEA0)\qquad(h)\ (0B0+)+(0B0-)+(yEB0)$

$(c)\ (xEB0)+(\pi E0)\qquad(d)\ (0B0+)+(0B0-)+(\pi E0)$

$\mathbf{(\pi Aq+)}\times(0A0\pm)=(\pi Aq\pm)\qquad(\pi Aq+)\times(0B0\pm)=(\pi Bq\pm)\qquad(\pi Aq+)\times(0Em\pm)=(\pi Eu\mp)$

$(\pi Aq+)\times(kEA0)=(zEAq)\qquad(\pi Aq+)\times(kEB0)=(zEBq)\qquad(\pi Aq+)\times(kGm)=(zGu)$

$(\pi Aq+)\times(\pi E0)=(0Eq)\qquad(\pi Aq+)\times(\pi Em\pm)=(0Eu\pm)\qquad(\pi Aq+)\times(0Eq)=(\pi E0)$

$(\pi Aq+)\times(kEAq)=(zEB0)\qquad(\pi Aq+)\times(kEBq)=(zEA0)\qquad(\pi Aq+)\times(\pi Aq+)=(0B0+)$

Table 4. (continued)

($\pi Aq-$) $\times(0A0\pm)=(\pi Aq\mp)$	$(\pi Aq-)\times(0B0\pm)=(\pi Bq\mp)$	$(\pi Aq-)\times(0Em\pm)=(\pi Eu\pm)$
$(\pi Aq-)\times(kEA0)=(zEAq)$	$(\pi Aq-)\times(kEB0)=(zEBq)$	$(\pi Aq-)\times(kGm)=(zGu)$
$(\pi Aq-)\times(\pi E0)=(0Eq)$	$(\pi Aq-)\times(\pi Em\pm)=(0Eu\mp)$	$(\pi Aq-)\times(0Eq)=(\pi E0)$
$(\pi Aq-)\times(kEAq)=(zEB0)$	$(\pi Aq-)\times(kEBq)=(zEA0)$	$(\pi Aq-)\times(\pi Aq\pm)=(0B0\mp)$
($\pi Bq+$) $\times(0A0\pm)=(\pi Bq\pm)$	$(\pi Bq+)\times(0B0\pm)=(\pi Aq\pm)$	$(\pi Bq+)\times(0Em\pm)=(\pi Eu\pm)$
$(\pi Bq+)\times(kEA0)=(zEBq)$	$(\pi Bq+)\times(kEB0)=(zEAq)$	$(\pi Bq+)\times(kGm)=(zGu)$
$(\pi Bq+)\times(\pi E0)=(0Eq)$	$(\pi Bq+)\times(\pi Em\pm)=(0Eu\mp)$	$(\pi Bq+)\times(0Eq)=(\pi E0)$
$(\pi Bq+)\times(kEAq)=(zEA0)$	$(\pi Bq+)\times(kEBq)=(zEB0)$	$(\pi Bq+)\times(\pi Aq\pm)=(0A0\pm)$
$(\pi Bq+)\times(\pi Bq+)=(0B0+)$		
($\pi Bq-$) $\times(0A0\pm)=(\pi Bq\mp)$	$(\pi Bq-)\times(0B0\pm)=(\pi Aq\mp)$	$(\pi Bq-)\times(0Em\pm)=(\pi Eu\pm)$
$(\pi Bq-)\times(kEA0)=(zEBq)$	$(\pi Bq-)\times(kEB0)=(zEAq)$	$(\pi Bq-)\times(kGm)=(zGu)$
$(\pi Bq-)\times(\pi E0)=(0Eq)$	$(\pi Bq-)\times(\pi Em\pm)=(0Eu\pm)$	$(\pi Bq-)\times(0Eq)=(\pi E0)$
$(\pi Bq-)\times(kEAq)=(zEA0)$	$(\pi Bq-)\times(kEBq)=(zEB0)$	$(\pi Bq-)\times(\pi Aq\pm)=(0A0\mp)$
$(\pi Bq-)\times(\pi Bq\pm)=(0B0\mp)$		

4. Discussion

In the presence of **L*n*** symmetry, the quasi-momentum $\hbar k$ and the quasi-angular momentum $\hbar m$ are conserved in scattering processes. Each of the line groups considered in this paper contains a subgroup of the form **L*n***, and thus the above conservation laws remain valid. The additional symmetry elements are $(\sigma_v|0)$ or $(\sigma_v|\frac{1}{2})$, which reverse the quasi-angular momentum, and $(U_d|0)$ which reverses both momenta. Hence energy eigenvalues are fourfold-degenerate, unless $k \triangleq -k$ (where '$\triangleq$' means 'equivalent', i.e. 'equal mod 2π') and/or $m \triangleq -m$ (where '$\triangleq$' means 'equal mod n').

Let $|\psi\rangle$, V and $|\phi\rangle$ belong to four-dimensional reps (kGm), $(k'Gm')$ and $(\kappa G\mu)$, respectively. Then the matrix element $\langle\phi|V|\psi\rangle$ must vanish unless $\kappa \triangleq k+k'$, $k-k'$, $-k+k'$ or $-k-k'$, and $\mu \triangleq m+m'$, $m-m'$, $-m+m'$ or $-m-m'$. In the special cases described above some of these values may coincide and then some two-dimensional or even one-dimensional reps are involved. On the other hand, these lower-dimensional reps possess additional quantum numbers, namely parities with respect to some involutions. When such an involution belongs to the line group under study, the corresponding parity is a good quantum number and therefore it has to be conserved too. Thus the parity with respect to $(\sigma_v|0)$ is conserved in the symmorphic line groups **L$\bar{n}$*m*** and **L($\overline{2n}$)2*m***, while the parity with respect to $(U_d|0)$ is conserved in all the line groups considered in this paper. (Notice, however, that the ± rep label does not always coincide with the latter parity, Božović and Vujičić 1981.)

In tables 3 and 4 some reduction coefficients are equal to two; in those (rare) cases the corresponding Clebsch–Gordan coefficients are not uniquely defined.

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